

**National Curriculum for**  
**GENERAL MATHEMATICS**  
**GRADES XI-XII**  
**2009**



**GOVERNMENT OF PAKISTAN**  
**MINISTRY OF EDUCATION**  
**ISLAMABAD**

# TABLE OF CONTENT

Introduction .....	1
CURRICULUM OF GENERAL MATHEMATICS FOR GRADE-XI .....	3
Unit 1    Applications to Algebraic Equations .....	3
Unit 2    Loans and Annuities .....	5
Unit 3    Islamic Banking .....	8
Unit 4    Purchasing and Inventory .....	10
Unit 5    Depreciation.....	12
Unit 6    Taxation .....	13
Unit 7    Linear Programming .....	15
Unit 8    Matrices and Determinants .....	16
Unit 9    Sequences and Series .....	18
Unit 10   Number Systems .....	19
CURRICULUM OF GENERAL MATHEMATICS FOR GRADE – XII .....	21
Unit 1    Introduction to Symbolic Package Maple .....	21
Unit 2    Functions and Graphs .....	22
Unit 3    Plane Analytic Geometry – Straight Line .....	24
Unit 4    Quadratic Equations.....	26
Unit 5    Partial Fractions .....	28
Unit 6    Permutation and Combination .....	29
Unit 7    Mathematical Induction and Binomial Theorem .....	30
Unit 8    Limit and Continuity.....	31
Unit 9    Differentiation.....	32
Unit 10   Application of Derivatives .....	34
Unit 11   Integration .....	35
Unit 12   Differential Equations.....	37
TEACHING STRATEGIES .....	38
Teaching Mathematics – Role of a Teacher.....	38
Effective Teaching Strategies.....	39
Time Distribution .....	40
ASSESSMENT AND EVALUATION.....	42
Assessment in Mathematics .....	42
The Traditional Examinations.....	43

TEACHING AND LEARNING RESOURCES .....	46
The Textbook .....	46
The Teacher's Manual .....	49
The Web-based Resources .....	50
CURRICULUM DEVELOPMENT TEAMS .....	51

## INTRODUCTION

The Curriculum for General Mathematics Grades XI and XII has been designed keeping in view the socio-economic, technical, professional and labour market needs of the country. It focuses on the contents of five mathematics standards framed in the National Curriculum of Mathematics 2006. The following themes permeate the curriculum for General Mathematics.

- The curriculum accentuates arithmetic skills that apply to both personal and vocational business opportunities.
- The financial and business Mathematics introduced in General Mathematics at Secondary School Certificate (SSC) level has been further extended to cover the topics like; loans and annuities, purchasing and inventory, depreciation, and taxation. They serve as a good foundation for further study in management, accounting and finance.
- Islamic Banking, being one of the emerging fields in global financial market, has a tremendous potential. The mission statement of State Bank of Pakistan also reads as; *to promote and develop Islamic banking industry in the line with best international practices, ensuring shariah compliance and transparency*. A blend of mathematics with the rudiments of Islamic banking has been introduced to steer the vision to develop a progressive and sound Islamic banking system in Pakistan.
- The curriculum stresses graphics that enabled the student to visualize and interpret mathematical expressions correctly rather to manipulate them 'blindly'.
- The curriculum integrates the use of appropriate technologies to enhance learning and doing mathematics.

The learning outcomes oriented National Curriculum of General Mathematics has been developed for humanitarian group.

Upon completion, the learner will be able to:

- recognize mathematics as integral to the development of different cultures and civilizations of the society.
- apply appropriate mathematics tools to solve real-life problems.
- examine real life situations by identifying mathematically valid arguments and drawing conclusion to enhance mathematical thinking.

At Higher Secondary School Certificate (HSSC) level, the following two combinations of elective subjects are well suited to the students who opt for Economics, Statistics and Computer Science at higher level.

- General mathematics, Statistics and Economics
- General mathematics, Statistics and Computer Science

In the curriculum, teacher's role has been re-routed that shifts from 'dispensing information' to planning investigative tasks, managing cooperative learning environment and supporting students' creativity in developing rational understanding of the concepts of Mathematics.

To ensure that assessment and evaluation are based on curriculum expectations and the achievement levels outlined in the curriculum, specific strategies are suggested that leads to the improvement of student learning. An effective learning-outcomes-oriented quality assurance system that is based on constant monitoring and effective feedback loops is recommended.

Print materials, particularly the textbooks, have to play a key role towards providing quality education at all levels. Although there are many stakeholders that contribute towards the overall learning of the student yet the importance of textbooks as a reservoir of information/knowledge cannot be ignored. In addition to the textbook, teaching and learning resources include teacher's manual and electronic resources. The guidelines to develop these resources are elaborated.

## CURRICULUM FOR GENERAL MATHEMATICS GRADE-XI

### UNIT 1 APPLICATIONS TO ALGEBRAIC EQUATIONS

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
1.1 Word Problems.	i) Know what the word problems are; ii) Identify the key words/phrases (plus, sum, more than, increased by, total, minus, difference, less than, less, decreased by, times, product, multiplied, of, twice, divided, quotient, is, equals) in word problems; iii) Translate key words and phrases into their algebraic equivalents; iv) Translate verbal statement to an algebraic statements; v) Set up the equation for a given word problem involving: <ul style="list-style-type: none"> <li>• One unknown quantity</li> <li>• Two unknown quantities</li> </ul> vi) Solve the word problems involving one unknown (in one equation) or two unknowns (in two equations). For example; <ul style="list-style-type: none"> <li>• The number problems</li> <li>• The mixture problems. (metals, chemical, products, medicines)</li> <li>• The investment problems. (interest, deposits variety of investments)</li> <li>• The motion problems (distance, speed and time)</li> </ul>
1.2 Ratio and Proportions	i) Recall the terms, ratio and proportions along with their notations; ii) Identify the terms:

	<ul style="list-style-type: none"> <li>• Antecedents</li> <li>• Consequent</li> <li>• Means</li> <li>• Extremes</li> </ul> <p>iii) Know the fundamental principle to solve problems of proportion that – the product of extremes is equal to the product of means;</p> <p>iv) Express the word problems involving:</p> <ul style="list-style-type: none"> <li>• Ratios</li> <li>• Direct Proportions</li> <li>• Inverse Proportions</li> <li>• Compound Proportions</li> </ul> <p>into linear equations and solve them</p>
1.3 Percentage	<p>i) Know:</p> <ul style="list-style-type: none"> <li>• The percentage as a fraction or ratio with a denominator 100</li> <li>• The base as the quantity one calculates a percent of the rate as the number of hundredths of the total (the base) under consideration</li> <li>• The percentage as the product of the rate times the base</li> </ul> <p>ii) Express the word problem involving:</p> <ul style="list-style-type: none"> <li>• Base, rate and percentage</li> <li>• Percentage increase or decrease</li> <li>• Profit or loss</li> <li>• Discount and commission</li> </ul> <p>into linear equations and solve them</p>

## UNIT 2 LOANS AND ANNUITIES

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
2.1 Introduction	Describe the terms borrowing and lending
2.2 Simple Interest	i) Define: <ul style="list-style-type: none"> <li>• The principal amount (P) as the amount borrowed/lent,</li> <li>• The term of loan (T) as length of time for which money is borrowed/lent,</li> <li>• The rate of interest (R) as the rate charged by the lender,</li> <li>• Simple interest (I) as the amount charged by the lender for utilizing the money</li> <li>• Amount due (A) as the total sum to be repaid by the borrower at the end of the term,</li> </ul> ii) Use appropriate formula to calculate: <ul style="list-style-type: none"> <li>• The amount of simple interest</li> <li>• The principal amount</li> <li>• The rate of interest</li> <li>• The term of loan</li> <li>• The amount due</li> </ul> iii) Use arithmetic sequence to calculate: <ul style="list-style-type: none"> <li>• The amount due (<math>n</math>-th term)</li> <li>• The amount at any stage of term of loan,</li> <li>• The term of loan (time duration)</li> </ul>
2.3 Compound Interest	i) Define/recognize: <ul style="list-style-type: none"> <li>• Compound interest</li> <li>• Interest period</li> <li>• Nominal rate</li> <li>• Effective rate</li> </ul>



	<p>ii) Use geometric sequence formula to calculate amount due when interest is compounded:</p> <ul style="list-style-type: none"> <li>• Annually</li> <li>• Semi annually</li> <li>• Quarterly</li> <li>• Monthly</li> </ul> <p>iii) Use geometric sequence formula to calculate:</p> <ul style="list-style-type: none"> <li>• Principal amount</li> <li>• Rate of compound interest</li> <li>• Term of loan</li> </ul>
2.4 Annuities	<p>i) Define:</p> <ul style="list-style-type: none"> <li>• Annuities as series of equal periodic payments or deposits with the interest on each one being compounded</li> <li>• Payment interval as the time between each payment</li> <li>• Ordinary annuity as an annuity where the periodic payments are made at the end of payment intervals</li> <li>• Annuity due as an annuity where the periodic payments are made at the beginning of the payments intervals</li> <li>• Terms of the annuity as the time from the beginning of the first payment interval to the end of last payment</li> <li>• Rent of annuity as the periodic payment</li> <li>• Present value of annuity as the lump sum of money invested today to receive a fixed sum of money at regular intervals</li> <li>• Future value of annuity as the sum of payments plus all interests earned at the end of the terms.</li> </ul> <p>ii) Calculate:</p> <ul style="list-style-type: none"> <li>• Present value of annuity</li> <li>• Future of annuity</li> </ul>

	<ul style="list-style-type: none"> <li>• Rent of annuity</li> <li>• Terms of annuity</li> </ul>
2.5 Mortgage Loans	<ul style="list-style-type: none"> <li>i) Explain mortgage loan</li> <li>ii) Calculate: <ul style="list-style-type: none"> <li>• Down payment</li> <li>• Principal amount</li> <li>• Rate of interest</li> <li>• Installment</li> <li>• Total repayable amount</li> <li>• Rebate in case of early repayment in mortgage problems</li> </ul> </li> </ul>

### UNIT 3 ISLAMIC BANKING

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
3.1 Introduction	Explain Islamic Banking
3.2 Musharaka	<ul style="list-style-type: none"><li>i) Define Musharaka as a relationship established by the parties through a mutual contract</li><li>ii) Know the principle of profit and loss sharing - profit is based on the agreement of the parties but loss is always subject to the ratio of investment</li><li>iii) Distribute the profit among the partners of Musharaka</li><li>iv) Calculate the loss to be shared by each partner of Musharaka</li><li>v) Distribute the assets and liabilities if the Musharaka is terminated</li></ul>
3.3 Mudarabah	<ul style="list-style-type: none"><li>i) Define Mudarabah as a kind of partnership where one partner gives money to another for investing it in a commercial enterprise</li><li>ii) Differentiate between Rabb-ul-mal and Mudarib</li><li>iii) Compare Musharaka and Mudarabah</li><li>iv) Distribute the profit between the partners of Mudarabah</li><li>v) Calculate the loss to be shared by each partner of Mudarabah</li><li>vi) Distribute the assets and liabilities, if the Mudarabah is terminated</li></ul>

<p>3.4 Murabahah</p>	<ul style="list-style-type: none"> <li>i) Define Murabahah.</li> <li>ii) Describe the only feature of Murabahah which distinguishes it from other kinds of sale.</li> <li>iii) Define sale in Shariah as — the exchange of a thing of a value by another thing of value with mutual consent.</li> <li>iv) Elaborate basic rules of sale which are related to the transaction of Murabahah as carried out by the financial institutions.</li> <li>v) Solve problems related to transaction of Murabahah.</li> </ul>
<p>3.5 Ijarah</p>	<ul style="list-style-type: none"> <li>i) Give the lexical meaning of Ijarah as — to give something on rent.</li> <li>ii) Define Ijarah as — to transfer the usufruct of a particular property to another person in exchange for a rent claimed from him.</li> <li>iii) Know that: <ul style="list-style-type: none"> <li>• The term Ijarah is analogous to the term leasing</li> <li>• The leaser is called Mujir</li> <li>• The lessee is called Mustajir</li> <li>• The rent payable to the lesser is called Ijarah</li> </ul> </li> <li>iv) Solve problems related with Ijarah</li> </ul>

## UNIT 4 PURCHASING AND INVENTORY

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
4.1 Introduction	Recognize: <ul style="list-style-type: none"> <li>• List price</li> <li>• Catalogue price</li> <li>• Marked price</li> <li>• Printed price</li> <li>• Invoice price</li> <li>• Extended price</li> <li>• Net price</li> </ul>
4.2 Trade Discount	i) Define: <ul style="list-style-type: none"> <li>• Trade discount</li> <li>• Simple discount</li> <li>• Several discounts (chain or series of discounts)</li> </ul> ii) Calculate: <ul style="list-style-type: none"> <li>• Simple discount</li> <li>• Rate of discount</li> <li>• Net price</li> <li>• List price</li> </ul> iii) Calculate several discounts and net price using: <ul style="list-style-type: none"> <li>• Step by step method</li> <li>• Single equivalent method</li> </ul>
4.3 Cash Discount	i) Define cash discount. ii) Calculate cash discount when net price and rate of discount are given. iii) Recognize the following terms of payment:

	<ul style="list-style-type: none"> <li>• Fixed term</li> <li>• End of month</li> <li>• Receipt of goods</li> </ul> <p>iv) Calculate cash discount and net price when terms of payment are given.</p>
4.4 Inventory	<p>i) Define merchandise inventory as the total value of the merchandise available for sale at any particular time.</p> <p>ii) Recognize the following:</p> <ul style="list-style-type: none"> <li>• Beginning inventory</li> <li>• Ending inventory</li> <li>• Sales (the total selling price of goods sold)</li> <li>• Gross profit (sales minus the cost of goods sold)</li> </ul> <p>iii) Calculate inventory cost when price of item(s) and quantity is given</p> <p>iv) Find inventory by retail inventory method.</p> <p>v) Calculate cost of goods sold.</p> <p>vi) Find inventory by gross profit method — the cost of ending inventory without a physical count of goods on hand.</p>

## UNIT 5 DEPRECIATION

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
5.1 Introduction	i) Define depreciation as an expense incurred due to the excessive use or obsolescence of certain asset. ii) Describe: <ul style="list-style-type: none"> <li>• Life of an asset as its useful life,</li> <li>• Salvage value of an asset as the value at the end of its useful life</li> </ul>
5.2 Calculation of Depreciation	i) Describe different methods to calculate depreciation. ii) Calculate depreciation by: <ul style="list-style-type: none"> <li>• Straight line method (the method which distributes total depreciation equally over the useful life of an asset)</li> <li>• Sum of the years method (the method which deducts an unequal amount of—depreciation each year)</li> <li>• Declining balance method (the method which calculates depreciation as rate times the book value of an asset at the beginning of each year)</li> <li>• Units of production method (the method which directly relates depreciation to amount of usage)</li> </ul>

## UNIT 6 TAXATION

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
6.1 Introduction	i) Define: <ul style="list-style-type: none"> <li>• Direct taxes (income tax, wealth tax, gift tax etc.)</li> <li>• Indirect taxes (sales tax, excise duty, customs duty etc.)</li> </ul>
6.2 Income Tax	i) Recognize: <ul style="list-style-type: none"> <li>• Financial year</li> <li>• Tax year</li> <li>• Minimum time scale</li> <li>• Basic pay</li> <li>• Allowances</li> <li>• Gross salary</li> <li>• Net salary</li> <li>• Income tax</li> <li>• Income tax deduction at source</li> <li>• Income tax return</li> </ul> ii) Calculate income of: <ul style="list-style-type: none"> <li>• A salaried person</li> <li>• Non-salaried person</li> </ul> iii) Calculate total taxable income of: <ul style="list-style-type: none"> <li>• A salaried person</li> <li>• Non-salaried person</li> </ul> iv) Compute income tax payable by <ul style="list-style-type: none"> <li>• A salaried person</li> <li>• Non-salaried person</li> </ul> with respect to appropriate slabs and rebates.



6.3 Sales Tax	<p>i) Define sales tax as a tax levied by the Federal Government on sales and supply of goods and services and on the goods imported into Pakistan.</p> <p>ii) Know that the following sectors are required to charge sales tax on their supplies/services:</p> <ul style="list-style-type: none"> <li>• Manufacturing</li> <li>• Import</li> <li>• Export</li> <li>• Services</li> <li>• Wholesale and retail stage</li> </ul> <p>iii) Solve real life problems involving Sales Tax.</p>
6.4 Federal Excise Duty	<p>i) Know that the Federal Excise Duty is payable on:</p> <ul style="list-style-type: none"> <li>• Goods produced/manufactured in Pakistan</li> <li>• Goods imported into Pakistan</li> <li>• Services rendered and provided in Pakistan</li> </ul> <p>ii) Solve real-life problems involving Federal Excise Duty</p>

## UNIT 7      LINEAR PROGRAMMING

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
7.1 Introduction	Define Linear Programming (LP) as planning of allocation of limited resources to obtain an optimal result.
7.2 Linear Inequalities	i) Find algebraic solutions of linear inequalities in one variable and represent them on number line. ii) Interpret graphically the linear inequalities in two variables. iii) Determine graphically the region bounded by simultaneous linear inequalities (up to three) of non-negative variables and shade the region bounded by them.
7.3 Feasible Region	i) Define: <ul style="list-style-type: none"> <li>• Linear Programming problem</li> <li>• Objective function</li> <li>• Problem constraints</li> <li>• Decision variables</li> </ul> ii) Define and show graphically the feasible region (or solution space) of an LP problem. iii) Identify the feasible region of simple LP problems.
7.4 Optimal Solution	i) Define optimal solution of an LP problem. ii) Find optimal solution (graphical) through the following systematic procedure: <ul style="list-style-type: none"> <li>• Establish the mathematical formulation of LP problem</li> <li>• Construct the graph</li> <li>• Identify the feasible region</li> <li>• Locate the solution points</li> <li>• Evaluate the objective function</li> <li>• Select the optimal solution</li> <li>• Verify the optimal solution by actually substituting values of variables from the feasible region</li> </ul> iii) Solve real life simple LP problems.

## UNIT 8 MATRICES AND DETERMINANTS

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
8.1 Matrices	i) Recall the concept of: <ul style="list-style-type: none"> <li>• A matrix</li> <li>• Order of a matrix</li> <li>• Equality of two matrices</li> </ul> ii) Define row matrix, column matrix, square matrix, rectangular matrix, zero/null matrix, identity matrix, scalar matrix, diagonal matrix, upper and lower triangular matrix, transpose of a matrix, symmetric matrix and skew-symmetric matrix.
8.2 Algebra of Matrices	i) Carryout scalar multiplication, addition/subtraction of matrices and multiplication of matrices with real entries. ii) Show that commutative property <ul style="list-style-type: none"> <li>• Holds under addition</li> <li>• Does not hold under multiplication, in general</li> </ul> iii) Verify that $(AB)^t = B^t A^t$
8.3 Determinants	i) Describe determinant of a square matrix, minor and cofactor of an element of a matrix. ii) Evaluate determinant of a square matrix using cofactors. iii) Define singular and non-singular matrices. iv) Know the adjoint of a square matrix. v) Use adjoint method to calculate inverse of a square matrix. vi) Verify the result $(AB)^{-1} = B^{-1} A^{-1}$

8.4 Properties of Determinants	<p>i) State the properties of determinants.</p> <p>ii) Evaluate the determinant without expansion (i.e. using properties of determinants).</p>
8.5 Solving System of Linear Equations	<p>i) Distinguish between homogeneous and non- homogeneous linear equations in 2 and 3 unknowns.</p> <p>ii) Solve a system of three homogeneous linear equations in three unknowns.</p> <p>iii) Define a consistent and inconsistent system of linear equations and demonstrate through examples.</p> <p>iv) Solve a system of 2-by-2 and 3-by-3 non-homogeneous linear equations using:</p> <ul style="list-style-type: none"> <li>• Matrix inversion method</li> <li>• Cramer's rule</li> </ul> <p>v) Use matrix inversion method and Cramer's rule to solve appropriate real life problems.</p>
<p><b>GUIDANCE FOR AUTHOR</b></p> <ul style="list-style-type: none"> <li>• Order of a matrix may be written as <math>m</math>-by-<math>n</math> instead of <math>m \times n</math>. See for reference. Golub, G.H. and Van Loan, C.F., 'Matrix Computations'. (3<sup>rd</sup> Edition). Johns Hopkins University Press, Baltimore USA (1996)</li> <li>• Order of a matrix should not exceed 3- by- 3</li> </ul>	

## UNIT 9 SEQUENCES AND SERIES

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
9.1 Sequence	i) Define a sequence (progression) and its terms. ii) Know that a sequence can be constructed from a formula or an inductive definition. iii) Recognize triangle, factorial and Pascal sequences.
9.2 Arithmetic Sequence	i) Define an arithmetic sequence. ii) Find the $n$ th or general term of an arithmetic sequence. iii) Solve problems involving arithmetic sequence.
9.3 Arithmetic Means	i) Know arithmetic means between two numbers. ii) Insert $n$ arithmetic means between two numbers.
9.4 Arithmetic Series	i) Define an arithmetic series. ii) Establish the formula to find the sum to $n$ terms of an arithmetic series. iii) Solve real life problems involving arithmetic series.
9.5 Geometric Sequence	i) Define a geometric sequence. ii) Find the $n$ th or general term of a geometric sequence. iii) Solve problems involving geometric sequence.
9.6 Geometric Means	i) Know geometric means between two numbers. ii) Insert $n$ geometric means between two numbers.
9.7 Geometric Series	i) Define a geometric series. ii) Find the sum of $n$ terms of a geometric series. iii) Find the sum of an infinite geometric series. iv) Convert the recurring decimal into an equivalent common fraction. v) Solve real life problems, involving geometric series.

## UNIT 10 NUMBER SYSTEMS

Contents/Scope		Learning Outcomes/ Skills
		The students will be able to:
10.1	Sets	i) Recall the sets denoted by N, W, Z, E, 0, P.
10.1.1	Operations on Sets	ii) Perform the following operations on sets: <ul style="list-style-type: none"> <li>• Union</li> <li>• Intersection</li> <li>• Difference</li> <li>• Complement</li> </ul>
10.1.2	Properties of Union and Intersection	iii) Verify the following fundamental properties for given sets: <ul style="list-style-type: none"> <li>• Commutative property of union</li> <li>• Commutative property of intersection</li> <li>• Associative property of union</li> <li>• Associative property of intersection</li> <li>• Distributive property of union over intersection</li> <li>• Distributive property of intersection over union</li> <li>• De morgan's laws</li> </ul>
10.1.3	Venn Diagram	iv) Use Venn diagram to represent: <ul style="list-style-type: none"> <li>• Union and intersection of sets</li> <li>• Difference of two sets</li> <li>• Complement of a set</li> </ul> v) Use Venn diagram to verify: <ul style="list-style-type: none"> <li>• Commutative law for union and intersection of sets</li> <li>• De Morgan's laws</li> </ul>
10.1.4	Ordered Pairs and Cartesian Product	vi) Recognize: <ul style="list-style-type: none"> <li>• Ordered pairs</li> <li>• Cartesian product</li> </ul>

10.2 Real Numbers	<ul style="list-style-type: none"> <li>i) Recall the set of real numbers as a union of sets of rational and irrational numbers.</li> <li>ii) Depict real numbers on the number line.</li> <li>iii) Demonstrate a number with terminating and non-terminating recurring decimals on the number line.</li> <li>iv) Give decimal representation of rational and irrational numbers.</li> </ul>
10.3 Properties of Real Numbers	Know the properties of real numbers.
10.4 Complex Numbers	<ul style="list-style-type: none"> <li>i) Recognize complex number <math>z</math> represented by an expression of the form <math>z=a+ib</math> or of the form <math>(a,b)</math> where <math>a</math> and <math>b</math> are real numbers and <math>i=\sqrt{-1}</math></li> <li>ii) Recognize <math>a</math> as real part of <math>z</math> and <math>b</math> as imaginary part of <math>z</math>.</li> <li>iii) Know the condition for equality of complex numbers.</li> <li>iv) Define <math>\bar{z}=a - ib</math> as the complex conjugate of <math>z=a+ib</math>.</li> <li>v) Define <math> z =\sqrt{a^2 + b^2}</math> as the absolute value or modulus of a complex number <math>z = a + ib</math>.</li> <li>vi) Carryout basic operations on complex numbers.</li> </ul>

**CURRICULUM OF GENERAL MATHEMATICS FOR GRADE – XII**  
**UNIT 1 INTRODUCTION TO SYMBOLIC PACKAGE: MAPLE**

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
1.1 Introduction	i) Recognize MAPLE environment. ii) Recognize basic MAPLE commands. iii) Use MAPLE as a calculator. iv) Use MAPLE help.
1.2 Polynomials	Use MAPLE commands for <ul style="list-style-type: none"> <li>• Factoring a polynomial</li> <li>• Expanding an expression</li> <li>• Simplifying an expression</li> <li>• Simplifying a rational expression</li> <li>• Substituting into an expression</li> </ul>
1.3 Graphics	i) Plot a two-dimensional graph. ii) Demonstrate domain and range of a plot. iii) Sketch parametric equations. iv) Know plotting options
1.4 Matrices	i) Recognize matrix and vector entry arrangement. ii) Apply matrix operations. iii) Compute inverse and transpose of a matrix.



## UNIT 2 FUNCTIONS AND GRAPHS

Contents/Scope	Learning Outcomes/ Skills
2.1 Function	<p>The students will be able to:</p> <ul style="list-style-type: none"> <li>i) Define a function and identify its domain and range.</li> <li>ii) Demonstrate the following:               <ul style="list-style-type: none"> <li>• Into function</li> <li>• One-one function</li> <li>• Into and one-one function (injective function)</li> <li>• Onto function (surjective function)</li> <li>• One-one and onto function (bijective functions)</li> </ul> </li> <li>iii) Examine whether a given relation is a function or not.</li> <li>iv) Define:               <ul style="list-style-type: none"> <li>• Open interval</li> <li>• Closed interval</li> <li>• Half open and half closed intervals</li> </ul> </li> <li>v) Identify through graph, the domain and range of a function.</li> <li>vi) Define and identify odd and even function.</li> <li>vii) Draw the graph of a modulus function (i.e., <math>y =  x </math>) and identify its domain and range.</li> </ul>
2.2 Composition of Functions	<ul style="list-style-type: none"> <li>i) Recognize the composition of functions.</li> <li>ii) Find the composition of two given functions.</li> </ul>
2.3 Graphical Representation of Functions	<ul style="list-style-type: none"> <li>i) Sketch the graphs of:               <ul style="list-style-type: none"> <li>• Linear functions (e.g. <math>y = ax + b</math>)</li> <li>• Non-linear functions (e.g. <math>y = x^2</math>)</li> </ul> </li> <li>ii) Sketch the graph of the function <math>y = x^n</math> where <math>n</math> is:               <ul style="list-style-type: none"> <li>• <math>a + ve</math> integer</li> <li>• <math>a - ve</math> integer (<math>x \neq 0</math>)</li> <li>• a rational number for <math>x &gt; 0</math></li> </ul> </li> <li>iii) Sketch graph of a quadratic function of the factor form <math>y = ax^2 + bx + c</math>, <math>a(\neq 0)</math>, <math>b, c</math> are integers</li> <li>iv) Sketch graph using factors.</li> <li>v) Predict functions from their graphs (use the form to</li> </ul>

	<p>predict the equation of a function of the type <math>f(x) = ax^2 + bx + c</math>, if two points where the graph crosses x-axis and third point on the curve, are given).</p> <p>vi) Display graphically:</p> <ul style="list-style-type: none"> <li>• The explicitly defined functions like <math>y = f(x)</math>, where <math>f(x) = e^x, e^{-x}, a^x, \log_a x, \log_e x</math></li> <li>• The implicitly defined functions such as <math>x^2 + y^2 = a^2</math> and <math>\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1</math></li> <li>• the parametric equations of functions such as <math>x = at^2, y = 2at</math></li> <li>• the functions of the type <math display="block">y = \begin{cases} x &amp; \text{when } 0 \leq x &lt; 1, \\ x - 1 &amp; \text{when } 1 \leq x \leq 2, \end{cases}</math></li> </ul> <p>vii) Use MAPLE graphic commands for two-dimensional plotting of:</p> <ul style="list-style-type: none"> <li>• An expression (or a function)</li> <li>• Parameterized form of a function</li> <li>• Implicit function by restricting domain and range.</li> </ul> <p>viii) Use MAPLE to plot different types of functions</p>
2.4 Application of Linear Functions	<p>i) Define the terms demand and supply.</p> <p>ii) Apply linear functions to represent demand (supply) as a function of price.</p> <p>iii) Know a consumption function as: how an economy can have persistent high unemployment or persistent high inflation.</p> <p>iv) Apply linear functions to represent consumption as function of disposable income.</p> <p>v) Define the terms cost, revenue and profit.</p> <p>vi) Apply linear functions to represent a cost, revenue and profit as a function of number of units produced and sold.</p>

### UNIT 3 PLANE ANALYTIC GEOMETRY – STRAIGHT LINE

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
3.1 Division of a Line segment	i) Recall distance formula to calculate distance between two points given in Cartesian plane ii) Find coordinates of a point that divides the line segment in a given ratio (internally and externally)
3.2 Slope of a Straight Line	i) Define the slope of a line. ii) Derive the formula to find the slope of a line passing through two points. iii) Find the condition that two straight lines with given slopes may be: <ul style="list-style-type: none"> <li>• Parallel to each other</li> <li>• Perpendicular to each other</li> </ul>
3.3 Equation of a Straight Line Parallel to Coordinate Axes	Find the equation of a straight line parallel to: <ul style="list-style-type: none"> <li>• <math>y</math>-axis and at a distance <math>a</math> from it</li> <li>• <math>x</math>-axis and at a distance <math>b</math> from it</li> </ul>
3.4 Standard Form of Equation of a Straight Line	i) Define intercepts of a straight line. Derive equation of a straight line in: <ul style="list-style-type: none"> <li>• Slope-intercept form</li> <li>• Point-slope form</li> <li>• Two-point form</li> <li>• Intercepts form</li> <li>• Symmetric form</li> </ul> ii) Show that a linear equation in two variables represents a straight line. iii) Reduce the general form of the equation of a straight line to the other standard forms.

3.5 Distance of a Point from a Line	Find the perpendicular distance from a point to the given straight line.
3.6 Concurrency of Straight lines	Find the condition of concurrency of three straight lines
3.7 Area of a Triangular Region	Find area of a triangular region whose vertices are given.

## UNIT 4 QUADRATIC EQUATIONS

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
4.1 Quadratic Equation	i) Define quadratic equation. ii) Solve a quadratic equation in one variable: <ul style="list-style-type: none"> <li>• By factorization</li> <li>• By method of completing the square</li> </ul>
4.2 Quadratic Formula	i) Use method of completing the square to derive quadratic formula. ii) Use quadratic formula to solve quadratic equation.
4.3 Equations Reducible to Quadratic Form	i) Solve equations, reducible to quadratic form, of the type $ax^4 + bx^2 + c = 0$ . ii) Solve the equations of the type $a p(x) + \frac{b}{p(x)} = c$ iii) Solve reciprocal equations of the type $a \left(x^2 + \frac{1}{x^2}\right) + b \left(x + \frac{1}{x}\right) + c = 0$ iv) Solve exponential equations in which the variables occur in exponents. v) Solve equations of the type $(x + a)(x + b)(x + c)(x + d) = k$ where $a + b = c + d$
4.4 Radical Equations	i) Solve equations of the type: <ul style="list-style-type: none"> <li>• <math>\sqrt{ax + b} = cx + d</math>,</li> <li>• <math>\sqrt{x + a} + \sqrt{x + b} = \sqrt{x + c}</math></li> <li>• <math>\sqrt{x^2 + px + m} + \sqrt{x^2 + px + n} = q</math></li> </ul>
4.5 Nature of the Roots of a Quadratic Equation	i) Define discriminant ( $b^2 - 4ac$ ) of the quadratic expression $ax^2 + bx + c$ ii) Find discriminant of a given quadratic equation

	<p>iii) Discuss the nature of roots of a quadratic equation through discriminant</p> <p>iv) Determine the nature of roots of a given quadratic equation and verify the result by solving the equation</p> <p>v) Determine the value of an unknown involved in a given quadratic equation when the nature of its roots is given.</p>
4.6 Roots and Coefficients of a Quadratic Equation	<p>i) Find the relation between the roots and the coefficients of a quadratic equation.</p> <p>ii) Find the sum and product of roots of a given quadratic equation without solving it.</p>
4.7 Formation of Quadratic Equation	<p>i) Establish and use the formula,</p> <ul style="list-style-type: none"> <li>• <math>x^2 - (\text{Sum of roots})x + (\text{Product of roots}) = 0</math>, to find a quadratic equation from the given roots.</li> </ul>
4.8 Simultaneous Equations	<p>i) Solve a system of two equations in two variables when</p> <ul style="list-style-type: none"> <li>• One equation is linear and the other is quadratic</li> <li>• Both the equations are quadratic</li> </ul> <p>ii) Solve the real life problems leading to quadratic equations.</p>

## UNIT 5 PARTIAL FRACTIONS

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
5.1 Proper, Improper and Rational Fraction	Define proper, improper and rational fraction.
5.2 Resolution of Fraction into Partial Fractions	i) Resolve an algebraic fraction into partial fractions when its denominator consists of: <ul style="list-style-type: none"> <li>• Non-repeated linear factors</li> <li>• Repeated linear factors</li> <li>• Non-repeated quadratic factors</li> <li>• Repeated quadratic factors</li> </ul> ii) Use MAPLE command to find partial fraction
<b>GUIDANCE FOR AUTHOR</b> In the denominator: <ul style="list-style-type: none"> <li>• For non-repeated linear case take only two/three factors</li> <li>• For non-repeated linear case take only two factors like <math>(x + a)^2 (x + b)</math></li> <li>• For non-repeated quadratic case take only two factors like <math>(x + a)(x + b)</math></li> <li>• For repeated quadratic case take only two factors <math>(x + a)(x^2 + b^2)</math></li> </ul>	

## UNIT 6 PERMUTATION AND COMBINATION

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
6.1 Factorial of a Natural Number	Know Kramp's factorial notation to express the product of first $n$ natural numbers by $n!$
6.2 Permutation	<p>i) Recognize the fundamental principle of counting and illustrate this principle using tree diagram.</p> <p>ii) Explain the meaning of permutation of <math>n</math> different objects taken <math>r</math> at a time and know the notation <math>{}^n P_r</math>.</p> <p>iii) Prove that <math>{}^n P_r = n(n-1)(n-2) \dots (n-r+1)</math> and hence deduce that:</p> <ul style="list-style-type: none"> <li>• <math>{}^n P_r = \frac{n!}{(n-r)!}</math></li> <li>• <math>{}^n P_n = n!</math></li> <li>• <math>0! = 1</math></li> </ul> <p>iv) Apply <math>{}^n P_r</math> to solve relevant problems of finding the number of arrangements of <math>n</math> objects taken <math>r</math> at a time (when all <math>n</math> objects are different and when some of them are alike).</p> <p>v) Find the arrangement of different objects around a circle.</p>
6.3 Combination	<p>i) Define combination of <math>n</math> different objects taken <math>r</math> at a time</p> <p>ii) Prove the formula and deduce that <math>{}^n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}</math></p> <ul style="list-style-type: none"> <li>• <math>\binom{n}{n} = \binom{n}{0} = 1</math></li> <li>• <math>\binom{n}{r} = \binom{n}{n-r}</math>, <math>\binom{n}{1} = \binom{n}{n-1} = n</math></li> <li>• <math>\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}</math></li> </ul> <p>iii) Solve problems involving combination</p> <p>iv) Use Maple command to find permutation and combinations</p>



## UNIT 7 MATHEMATICAL INDUCTION AND BINOMIAL THEOREM

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
7.1 Mathematical Induction	<ul style="list-style-type: none"> <li>i) Describe principle of Mathematical Induction.</li> <li>ii) Apply the principle to prove the statements, identities or formulae.</li> </ul>
7.2 Binomial Theorem	<ul style="list-style-type: none"> <li>i) Use Pascal's triangle to find the expansion of <math>(x + y)^n</math> where <math>n</math> is a small positive integer.</li> <li>ii) State-binomial theorem for positive integral index.</li> <li>iii) Expand <math>(x + y)^n</math> using binomial theorem and find its general term.</li> <li>iv) Find the specified term in the expansion of <math>(x + y)^n</math></li> </ul>
7.3 Binomial Series	<ul style="list-style-type: none"> <li>i) Expand <math>(1 + x)^n</math> where <math>n</math> is a positive integer and extend this result for all rational values of <math>n</math>.</li> <li>ii) Expand <math>(1 + x)^n</math> in ascending powers of <math>x</math> and explain its validity/convergence for <math> x  &lt; 1</math> where <math>n</math> is a rational number</li> <li>iii) Determine the approximate values of the binomial expansion having indices as -ve integers or fractions</li> </ul>

## UNIT 8 LIMIT AND CONTINUITY

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
8.1 Limit of a Function	i) Identify a real number by a point on the number line. ii) Represent: <ul style="list-style-type: none"> <li>• Open interval</li> <li>• Closed interval</li> <li>• Half open and half closed intervals on the number line.</li> </ul> iii) Explain the meaning of phrase: <ul style="list-style-type: none"> <li>• <math>x</math> tends to zero (<math>x \rightarrow 0</math>)</li> <li>• <math>x</math> tends to <math>a</math> (<math>x \rightarrow a</math>)</li> <li>• <math>x</math> tends to infinity (<math>x \rightarrow \infty</math>)</li> </ul> iv) Define limit of a sequence. v) Find the limit of a sequence whose $n$ th term is given. vi) Define limit of a function. vii) State the rules on limits of sum, difference, product and quotient of functions and demonstrate through examples.
8.2 Important Limits	i) Evaluate the limits of the functions of following types: <ul style="list-style-type: none"> <li>• <math>\frac{x^n - a^n}{x - a}, \frac{x - a}{\sqrt{x} - \sqrt{a}}</math> when <math>x \rightarrow a</math></li> <li>• <math>\left(1 + \frac{1}{x}\right)^x</math> when <math>x \rightarrow \infty</math></li> <li>• <math>\frac{\sqrt{x+a} - \sqrt{a}}{x}, (1+x)^{\frac{1}{x}}</math> when <math>x \rightarrow 0</math></li> </ul> ii) Evaluate limits of different algebraic and exponential functions iii) Use MAPLE command to evaluate limit of a function
8.3 Continuous and Discontinuous Functions	i) Define continuity of a function at a point and in an interval ii) Test continuity and discontinuity of a function at a point and in an interval iii) Use MAPLE command to test continuity of a function at a point and in a given interval

## UNIT 9 DIFFERENTIATION

Contents/Scope	Learning Outcomes/ Skills
9.1 Derivative of a Function	<p>The students will be able to:</p> <ul style="list-style-type: none"> <li>i) Distinguish between independent and dependent variable.</li> <li>ii) Estimate corresponding change in the dependent variable when independent variable is incremented (decremented).</li> <li>iii) Explain the concept of rate of change.</li> <li>iv) Define derivative of a function as an instantaneous rate of change of a variable with respect to another variable.</li> <li>v) Define derivative or differential coefficient of a function.</li> <li>vi) Apply the following formulae for differentiation:               <ul style="list-style-type: none"> <li>• <math>\frac{dx^n}{dx} = nx^{n-1}</math></li> <li>• <math>\frac{d}{dx} (ax+b)^n = na(ax+b)^{n-1}</math></li> </ul>               where <math>n</math> is a real number.             </li> </ul>
9.2 Rules of Differentiation	<p>State the following rules of differentiation.</p> <ul style="list-style-type: none"> <li>• The derivative of a constant is zero</li> <li>• The derivative of any constant multiple of a function is equal to the product of that constant and the derivative of the function</li> <li>• The derivative of a sum (or difference) of two functions is equal to the sum (or difference) of their derivatives</li> <li>• The derivative of a product of two functions is equal to (the first function) x (derivative of the second function) plus (derivative of the first function) x (the second function)</li> <li>• The derivative of a quotient of two functions is equal to denominator times the derivative of the numerator, minus the numerator times the derivative of the</li> </ul>

	denominator, all divided by the square of the denominator
9.3 Application of Rules of Differentiation	<p>Differentiate:</p> <ul style="list-style-type: none"> <li>• Constant multiple of <math>x^n</math></li> <li>• Sum (or difference) of functions</li> <li>• Polynomials</li> <li>• Product of functions</li> <li>• Quotient of two functions</li> </ul>
9.4 Chain Rule	<p>i) Apply <math>\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}</math> when <math>y=f(u)</math> and <math>u = g(x)</math></p> <p>ii) Use chain rule to show that:  <math display="block">\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} f'(x)</math></p> <p>iii) Prove that:  <math display="block">\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}</math> when <math>y = f(t)</math> and <math>x = g(t)</math></p> <p>iv) Find derivative of an implicit function.</p>
9.5 Differentiation of Exponential and Logarithmic Functions	<p>i) State and apply the formulae to find derivatives of <math>e^x</math>, <math>a^x</math>, <math>\ln x</math> and <math>\log_a x</math></p> <p>ii) Use logarithmic differentiation to find derivative of algebraic expressions involving product, quotient and power</p>
	Use MAPLE command to differentiate a function

## UNIT 10 APPLICATION OF DERIVATIVES

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
10.1 Higher Order Derivatives	i) Find higher order derivatives of algebraic, exponential and logarithmic functions. ii) Find the second derivative of implicit and parametric functions. iii) Use MAPLE command to find higher order derivative of a function.
10.2 Application of Derivatives	i) Give geometrical interpretation of derivative. ii) Find the equation of tangent and normal to the curve at a given point.
10.3 Maxima and Minima	i) Define increasing and decreasing functions. ii) Prove that: <ul style="list-style-type: none"> <li>• <math>f(x)</math> is increasing if <math>f'(x) &gt; 0</math></li> <li>• <math>f(x)</math> is decreasing if <math>f'(x) &lt; 0</math></li> </ul> where $f(x)$ is a differentiable function in a given interval. iii) Define critical points for a function in a given domain. iv) Examine a given function for extreme (maximum or minimum) values. v) State the second derivative rule to find the extreme value of a function at a critical point. vi) Use second derivative rule to examine a given function for extreme values vii) Solve real life problems related to maximum or minimum values viii) Use MAPLE command to find maximum (minimum) value of a function

## UNIT 11 INTEGRATION

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
11.1 Introduction	i) Know the concept of the integral as an accumulator. ii) Know integration as inverse process of differentiation. iii) Explain constant of integration. iv) Know simple standard integrals which directly follow from standard differentiation formulas.
11.2 Rules of Integration	i) Recognize the following rules of integration: <ul style="list-style-type: none"> <li>• The integral of the product of a constant and a function is the product of the constant and the integral of the function</li> <li>• The integral of the sum (difference) of a finite number of functions is equal to the sum (difference) of their integrals</li> </ul> ii) Use appropriate formulas to evaluate various integrals.
11.3 Integration by Substitution	i) Explain the method of integration by substitution. ii) Recognize the following expressions for integrals: <ul style="list-style-type: none"> <li>• <math>\int f(g(x)) \cdot g(x) dx</math></li> <li>• <math>\int [f(x)]^n f(x) dx</math></li> <li>• <math>\int \frac{f'(x)}{f(x)} dx</math></li> <li>• <math>\int e^{ax} [af'(x) + f'(x)] dx</math></li> </ul> iii) Apply method of substitution (algebraic only) to evaluate indefinite integrals.
11.4 Integration by Parts	i) Recognize the formula for integration by parts. ii) Evaluate integrals using integration by parts (simple cases only).

<p>11.5 Integration using Partial Fractions</p>	<p>Use partial fractions to find <math>\int \frac{f(x)}{g(x)} dx</math>, where <math>f(x)</math> and <math>g(x)</math> are algebraic functions such that <math>g(x) \neq 0</math></p>
<p>11.6 Definite Integrals</p>	<p>i) Define definite integral as the limit of a sum.</p> <p>ii) Describe the fundamental theorem of integral calculus and recognize the following basic properties:</p> <ul style="list-style-type: none"> <li>• <math>\int_a^b f(x)dx = g(x) \Big _a^b = g(b) - g(a)</math></li> <li>• <math>\int_a^a f(x)dx = 0</math></li> <li>• <math>\int_a^b f(x) dx = \int_a^b f(y) dy</math></li> <li>• <math>\int_a^b f(x) dx = - \int_b^a f(y) dy</math></li> <li>• <math>\int_a^b f(x)dx = \int_a^c f(y)dx + \int_c^b f(x)dx, a &lt; c &lt; b</math></li> <li>• <math>\int_a^a f(x)dx = \begin{cases} 2 \int_0^a f(x)dx &amp; \text{when } f(-x) = f(x) \\ 0 &amp; \text{when } f(-x) = -f(x) \end{cases}</math></li> </ul> <p>iii) Extend techniques of integration using properties to evaluate definite integrals.</p> <p>iv) Represent definite integral as the area under the curve.</p> <p>v) Apply definite integrals to calculate area under the curve</p> <p>vi) Use MAPLE command to evaluate definite and indefinite integrals</p>

## UNIT 12 DIFFERENTIAL EQUATIONS

Contents/Scope	Learning Outcomes/ Skills
	The students will be able to:
12.1 Differential Equation	Define: <ul style="list-style-type: none"><li>• Ordinary Differential Equation (DE)</li><li>• Order of a DE</li><li>• Degree of a DE</li><li>• Solution of a DE – general solution and particular solution</li></ul>
12.2 Formation of Differential	Demonstrate the concept of formation of Equation differential equation.
12.3 Solution of Differential Equation	i) – Solve differential equations of first order and first degree by the method of separation of variables. ii) Solve real life problems related to differential equations.



# TEACHING STRATEGIES

## Introduction

---

In the classroom, teachers transmit textbook facts to students, who in return are expected to memorize and regurgitate them in examinations. The teachers are so ingrained that they find this method of lecture and recitation as a good way of teaching a large number of students in their classrooms. In Mathematics, students memorize rules without understanding their rationale. There is no doubt that the timely reward to this way is more immediate and more apparent but this instrumental learning does not bring desired result subsequently. The memorized rules may work for a limited range of similar problems. But students do not feel comfortable when they face different or challenging tasks. Consequently, the students are totally dependent upon teachers. They cannot progress in thinking hence their self-esteem is low.

To capture all aspects of expertise, competence, knowledge and facility which are necessary for anyone to learn Mathematics, Kilpatrick et al\* (2001) present the notion of mathematical proficiency that is composed of following five interwoven but interdependent strands:

- **Conceptual understanding** – comprehension of mathematical concepts, operations and relations.
- **Procedural fluency** – skill in carrying out procedures flexibly, accurately, efficiently and appropriately.
- **Strategic competence** – ability to formulate, represent and solve mathematical problems.
- **Adaptive reasoning** – capacity for logical thought, reflection, explanation and justification.
- **Productive disposition** – habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy.

## Part I Teaching Mathematics – Role of a Teacher

---

Research indicates that teachers who have good background in Mathematics also add richness to their lessons, involves students extensively in mathematical dialogue and capitalize on students' questions/discussions to weave/extend mathematical relationship. They do not list only the definitions and step-by-step procedures for students to memorize without understanding their meaning and function

Teachers need to assume a new role, if students are to construct their own mathematical understanding. Rather than just pouring mathematical knowledge into students' heads, teacher must create a stimulating environment that encourages mathematical learning through increasing interactivity.

---

\* Kilpatrick, J., Swafford, J., and Findell, B. (Eds) (2001) Adding It Up: Helping Children Learn Mathematics, Mathematics Learning Study Committee, Centre for Education, Washington, DC: National Academies Press.

Teaching in Mathematical classroom requires listening to the students, understanding their level of thinking, setting a task and analyzing outcomes of the tasks in order to understand how student construct meanings – this is contrary to a traditional way of teaching. The teachers’ role shifts from dispensing information to planning investigative tasks, managing a cooperative learning environment and supporting students’ creativity in developing rational understanding concepts. This improved teaching practice should include the following aspects of a teacher's role. The teacher should be:

- A planner of practical tasks for the students to consolidate and organize their informal knowledge
- An organizer of the establishment of mathematical tasks in the classroom so that the students can work in a social setting and develop rational understanding
- An encourager who asks questions, support and develops students’ mathematical thinking and communication
- A negotiator helping students to discuss various meanings/solutions of a concept/question and to achieve a common agreement
- A mediator supporting the establishment of an environment where students express opinions and experiences in the classroom equally

Thus a teacher's primary responsibilities are to assist learners’ cognitive reconstruction and conceptual re-organization through providing them the opportunities for interaction in mathematical tasks that encourage discussion and negotiation of ideas to help them to develop conceptual understanding.

## **Part II Effective Teaching Strategies**

---

Students learn things in many different ways. They do not always learn best by sitting and listening to the teacher. They can learn by presentation and explanation by the teacher, consolidation and practice, practical work/problems and investigating mathematics.

### **Investigating Mathematics**

Teachers may set students a challenge, matched to their ability, which leads them to discover and practice some new mathematics for themselves. The key point about investigations is that students are encouraged to make their own decisions about:

- Where to start
- How to deal with challenges
- What mathematics they need to use
- How they can communicate this mathematics
- How to describe what they have discovered

### **Problem Solving**

A problem is a statement or proposition requiring an algebraic, geometric, or other mathematical solution. A widespread opinion that problem solving should be the central focus of the curriculum for mathematics strengthens the fact that ‘learning to solve problems is the principal reason for studying mathematics’.

A problem exists when there is a situation a learner wants to resolve but no solution is readily apparent. For example Shahzaib counted 19 cycle-wheels, run by 7 cycle-riders, going past his house. How many tricycles were there? Working on this problem offers a good practice in addition, multiplication and division skills. But the important goal of this problem is to help students think systematically about possibilities and record thinking.

Research suggests that a problem solver needs to become better acquainted with a problem and works for a clearer understanding of it before progressing towards a solution. The path from understanding the problem to devising a plan may sometimes be long but experience and practice are the best teachers to contrive. The plan gives a general outline of direction to solve the problem. Having arrived at a result, it is verified by referring back to the original problem.

### **Part III** Time Distribution

Teaching schedules are among the integral parts of Mathematics classrooms. They help management to run and monitor the teaching of a particular subject. The following tables indicating unit-wise time distribution will be supportive to the teachers and education planners.

#### UNIT-WISE TIME DISTRIBUTION – GRADE XI

Unit	Title	Periods (40 minutes each)
1.	Applications to Algebraic Equations	24
2.	Loans and Annuities	24
3.	Islamic Banking	21
4.	Purchasing and Inventory	24
5.	Depreciation	12
6.	Taxation	21
7.	Linear Programming	14
8.	Matrices and Determinants	21
9.	Sequences and Series	21
10.	Number Systems	18
<b>TOTAL</b>		<b>200</b> (8 periods per week)

## UNIT-WISE TIME DISTRIBUTION – GRADE XII

Unit	Title	Periods <small>(40 minutes each)</small>
1.	Introduction to Symbolic Package Maple	21
2.	Function and Graphs	24
3.	Plane Analytic Geometry – Straight Line	21
4.	Quadratic Equations	21
5.	Partial Fractions	7
6.	Permutation and Combination	18
7.	Mathematical Induction and Binomial Theorem	18
8.	Limit and Continuity	15
9.	Differentiation	18
10.	Application of Derivatives	15
11	Integration	15
12	Differential Equations	7
<b>TOTAL</b>		<b>200</b> <small>(8 periods per week)</small>

# ASSESSMENT AND EVALUATION

## Introduction

---

Assessment is the process of gathering information using a variety of tools and techniques that reflect how well a student is achieving the curriculum expectations in a subject. As part of assessment teachers provide students with descriptive feedback that guides their efforts towards improvement. The quality of assessment largely determines the quality of evaluation. Evaluation refers to the process of judgments and decisions based on the interpretation of evidence gathered through assessment.

Rowntree<sup>†</sup> (1990) defined assessment as having two purposes: firstly to support and provide feedback to learners and improve their ongoing learning, and secondly to report on what they had already achieved. In essence the first is formative assessment and the second is summative assessment. Morgan and O'Reilly<sup>‡</sup> (1999) believe that assessment is the engine that drives and shapes learning rather than an end of course event that gives grades and reports on performance.

Assessment and evaluation should be based on curriculum expectations and the achievement levels outlined in the national curriculum. To ensure that assessment and evaluation lead to the improvement of student learning, teachers must use specific assessment and evaluation strategies that:

- Address both what students learn and how well they learn
- Are administered over a period of time and designed to provide opportunities for students to demonstrate full range of their learning
- Ensure that each student is given clear directions for improvement
- Promote students' ability to assess their own learning
- Are communicated clearly to students and parents in advance

## Part I Assessment in Mathematics

---

It should be kept in mind that in Mathematics a single type of assessment can frustrate students, diminish their self-confidence and make them feel anxious about the subject. In reality the understanding of mathematical concepts encompasses a broad range of abilities. Examples of various templates to assess different abilities are mentioned below.

Assessment must include by focusing on student's ability to:

- Communicate mathematically
- Reason, analyze, to think and act in positive ways
- Comprehend the key concepts
- Evaluate the effectiveness of using different strategies to address the same problem

---

<sup>†</sup> Rowntree, D. (1990) Teaching through Self-Instruction (Second Ed). London: Kogan Page.

<sup>‡</sup> Morgan, C. and O'Reilly, M. (1999) Assessing Open and Distance Learners, London: Kogan Page.

- Use a variety of strategies to problem solving and to make mathematical connections
- Discriminate between relevant and irrelevant attributes of a concept in selecting examples
- Integrate and to make sense of mathematical concepts and procedure
- Examine real life situations by reasoning mathematically

Learning of mathematics, being a cumulative process, occurs as experiences contribute to understanding. Suggested below are the assessment strategies to obtain valid and reliable picture of students' understanding and achievement.

- Classroom-based assessments** that include anecdotal records, checklists, rating scales, portfolios peer- and self-assessment.
- Teacher-designed test formats** that include oral examination, assignments, short answers, matching, multiple-choice, fill-in and true-false items.

## **Part II    The Traditional Examinations**

---

For assessment and evaluation of grades below secondary level, the institutions adopt their own criteria. The means by which each institution achieves quality should differ according to the circumstances in which it operates, but each must give priority to meeting students' expectations in terms of learning outcomes they can legitimately expect to achieve. In essence an effective learning-outcomes-oriented quality assurance system must be based on constant monitoring and effective feedback loops.

Bearing in mind the requirement of simplicity in considering assessment strategies, the examinations in traditional paper-based mode with place and time-specific activities, are easy to organize for institutions (Boards of Intermediate and Secondary Education). When a formal examination, for Secondary School Certificate (SSC) or Higher Secondary School Certificate (HSSC), is used for assessment there are examination centers, infrastructure to supply and secure examination papers before examination and arrangements to check the identities of the candidates, invigilate the examination and collect the scripts for marking. Marks are then gathered and results are published in a timely manner.

### **Instructions for HSSC Examinations**

The examining institutions or bodies including all Boards of Intermediate and Secondary Education for the conduct of HSSC examination in the subject of General Mathematics should follow instructions as given below.

(1) The question papers should be balanced in all respect. Following table, showing weightage difficulty level of questions, is suggested to be a practicable criterion for a balanced question paper of Mathematics.

Difficulty Level of Questions	Weightage
Easy	15 %
Average	70 %
Difficult	15 %

(2) Two papers (one for HSSC Part-I and the other for HSSC Part-II), each carrying 100 marks, are proposed for the subject of General Mathematics.

(3) The examiners will set the papers keeping in view the tables (unit-wise weightages at pages 40, 41).

(4) To integrate technology the symbolic package MAPLE has been introduced first time. No question will be set from unit (Introduction to Symbolic Package: MAPLE) until the instructors are trained and institutions are equipped appropriately within next three academic years. Zero weightage has therefore been assumed for the said unit initially.

#### UNIT WISE WEIGHTAGE – PAPER (GRADE – XI)

Unit	Title	Weightage
1	Applications to Algebraic Equations	10%
2	Loans and Annuities	15%
3	Islamic Banking	8%
4	Purchasing and Inventory	8%
5	Depreciation	5%
6	Taxation	10%
7	Linear Programming	6%
8	Matrices and Determinants	15%
9	Sequences and Series	15%
10	Number Systems.	8%
TOTAL		100%

UNIT WISE WEIGHTAGE –PAPER (GRADE-XII)

Unit	Title	Weightage
1	Introduction to Symbolic Package: Maple	0%
2	Functions and Graphs	12%
3	Plane Analytic Geometry — Straight Line	12%
4	Quadratic Equations	10%
5	Partial Fractions	4%
6	Permutation and Combination	10%
7	Mathematical Induction and Binomial Theorem	10%
8	Limit and Continuity	10%
9	Differentiation	10%
10	Application of Derivatives	6%
11	Integration	10%
12	Differential Equations	6%
<b>TOTAL</b>		<b>100%</b>



# TEACHING AND LEARNING RESOURCES

## Introduction

---

The government prescribed textbook is the only teaching and learning tool used in most of the Pakistani institutions. Though many other resources are also available, accessible and affordable yet teachers rarely use them to support the learning. In addition to the textbook, the teaching and learning resources include teacher's manual, workbook and electronic resources.

## Part I The Textbook

---

There are many important entities involved to revamp the entire education system. The institution has to play its own role, parents have to contribute their share and teachers have to assume a significant place in fostering education. Print materials, particularly the textbooks, have to play a key role towards providing quality education at all levels. Although there are many stakeholders that contribute towards the overall learning of the student yet the importance of textbook as a reservoir of information/ knowledge cannot be ignored.

Textbook writers have a vital role to play in penetrating the young minds through their writing. A textbook:

- Whose content as well as presentation is thoughtfully planned
- Which is written by qualified and competent subject expert(s) and
- Which is attractive and engaging

must stimulate the interest of teacher and the taught

### Guidelines for Textbook Authors

Textbooks aimed at lower level tend to include more learning features than those at richer level. However in textbook writing generally the following aspects may be taken into consideration.

- The textbook should be in line with the objectives of National Curriculum
- The author should bring himself to the mental level of students he is writing for
- The span of the textbook should be fairly reasonable
- The material should not be cramped. To make it more digestible, it may be chunked into smaller parts with headings
- The textbook is expected to provide accurate and up-to-date information
- The text material should be arranged in a logical manner; simple to complex, familiar to unfamiliar and concrete to abstract

- The text material must be free from ambiguities and errors (both mathematical and typographical)
- The content provided in the textbook should not develop wrong concepts
- The text should be clear and concise. It should not give any other meaning than the one intended
- Special attention should be paid to geometrical portions. Every table, line drawing and graph should be labeled appropriately
- Footnotes and side notes may be inserted wherever necessary

## Textbook Style and Structure

To make a textbook an effective teaching and learning tool its style and structure is given due importance. The material needs to be structured in a coherent and logical way and that writing style should be reader friendly.

Unit Opening	
Unit Outline	Include list of headings.
Student Learning Outcomes (SLOs)	One SLO for each heading may be included. If they are numerous then a reasonable number is acceptable
Real Life Relevance	Illustrate the real life relevance of the unit if possible.
Short Introduction	Explain what this unit covers and why.

Unit Body	
Key Terms.	Use italics for emphasis and bold for key terms. Define key terms when first introduced and collate them with their definitions in the glossary.
Running Glossary	Key terms and definitions may be pulled out from the main body of text so that students spot them easily in the unit body (e.g. in the margins).
Feature Boxes	Regular feature boxes may include various contents such as a mathematical formula, a working rule or a statement of theorem.
Illustrative Examples	Include illustrative examples to develop conceptual understanding of the topic.
Problem Sets	Special attention should be paid on preparation of Problem sets. Correlate Mathematics with real life situations and include sufficient

	<p>exercises on real life problems almost in every problem set, if appropriate.</p> <p>The questions on the application of Mathematics in other fields of study are also very useful.</p>
Learning Review Points	Include bulleted questions for students to check their understanding at regular intervals. Possible labels include 'self-test points' or 'checkpoints'
Tips or Hints	Separated from the main body of text, they allow the author to speak directly to the student, offering useful advice or flagging important points
Visuals	Tables, graphs, line Drawings and lists may be used to break up the text

Unit Binding	
Problem Set (Review)	Include multiple choice questions, interpretive exercises and fill-in items. Students may also be asked to label diagrams or write a one word answer to short question.
Summary	Include a review of the main concepts. This can relate to the SLOs by covering each in turn (bullets points work well). The summary should not include any new information.

End of Textbook	
Glossary	Include only the key terms in the glossary.
Answers to Problems	Include answers to the problem sets unit wise.
Appendices	Include extra information, the student needs such as list of mathematical formulas, log tables and relevant websites.
Bibliography	Include bibliography and list of books for suggested reading where appropriate.
Index	Include index for the key terms used in the book.

## **Part II** The Teacher's Manual

---

Ideally the teacher's manual should come with the textbook. The manual is aimed at informing teachers how the textbook is written and how best to use it to facilitate student learning. It can be seen as a mean of helping teachers develop professionally. It provides detailed explanation of key concepts and the way to teach a particular topic. Its basic features are as below.

The teacher's manual should:

- Be easy to understand and use
- Include teaching learning resources
- Establish a question bank (having questions different from text) and suggest interactive quizzes corresponding to each unit
- Involve various up-to-date and relevant teaching strategies and rationale for suggested teaching
- Identify resources needed for teaching strategies
- Explain how to implement each teaching strategy
- Expand and develop teacher's repertoire of knowledge and skills
- Identify assessment strategies

### Part III The Web-based Resources

The use of World Wide Web (www) is growing very fast to access an immense volume of rapidly evolving information. It is acting as a driving force since its ease of use makes the internet trivially accessible to the students even with a little knowledge of computer. Through web-based links as mentioned below the teachers, parents and students will be able to:

- Access various sites of Mathematics around the world
- View three-dimensional figures, graphics, lesson plans, activities and various books of interest

Title of Website	Universal Resource Locator (URL)
A+Math	<a href="http://www.aplusmath.com/">http://www.aplusmath.com/</a>
AAA Math	<a href="http://www.aaamath.com/">http://www.aaamath.com/</a>
Academic Info- Mathematics	<a href="http://www.academicinfo.net/math.html">http://www.academicinfo.net/math.html</a>
Algebra Buster	<a href="http://www.algebra-online.com/">http://www.algebra-online.com/</a>
Algebra Helper	<a href="http://www.algebrahelp.com/index.jsp">http://www.algebrahelp.com/index.jsp</a>
Class Zone	<a href="http://www.classzone.com/math_middle.cfm">http://www.classzone.com/math_middle.cfm</a>
Click on Bricks	<a href="http://kathyschrock.net/clickonbricks/index2.htm">http://kathyschrock.net/clickonbricks/index2.htm</a>
Cool Math	<a href="http://www.coolmath.com/">http://www.coolmath.com/</a>
Frank Potter's Science Gems-Mathematics	<a href="http://www.sciencegems.com/math.html">http://www.sciencegems.com/math.html</a>
Funbrain	<a href="http://www.funbrain.com/numbers.html">http://www.funbrain.com/numbers.html</a>
Geometry	<a href="http://www.mathleague.com/help/geometry/geometry.htm">http://www.mathleague.com/help/geometry/geometry.htm</a>
Internet Mathematics Library	<a href="http://www.mathforum.org/library">http://www.mathforum.org/library</a>
MAPLE	<a href="http://www.maplesoft.com">http://www.maplesoft.com</a>
Math Archives	<a href="http://www.archives.math.utk.edu/">http://www.archives.math.utk.edu/</a>
Math Goodies	<a href="http://www.mathgoodies.com">http://www.mathgoodies.com</a>
Math World	<a href="http://www.mathworld.wolfram.com">http://www.mathworld.wolfram.com</a>
Math2	<a href="http://www.math2.org/">http://www.math2.org/</a>
MATHEMATICA	<a href="http://www.wolfram.com/products/mathematica/index.html">http://www.wolfram.com/products/mathematica/index.html</a>
Mathematical Interactivities	<a href="http://mathematics.hellam.net/">http://mathematics.hellam.net/</a>
Mega Mathematics	<a href="http://www.c3lanl.gov/mega-math/">http://www.c3lanl.gov/mega-math/</a>
Purplemath	<a href="http://www.purplemath.com/internet.htm">http://www.purplemath.com/internet.htm</a>
S.O.S. Mathematics	<a href="http://www.sosmath.com">http://www.sosmath.com</a>
Teaching madeEasier	<a href="http://www.teachingmadeeasier.com/math.html">http://www.teachingmadeeasier.com/math.html</a>
The MathWorks (MATLAB)	<a href="http://www.mathworks.com">http://www.mathworks.com</a>
Webmath	<a href="http://www.webmath.com/">http://www.webmath.com/</a>

## General Mathematics Curriculum Development Committee

### Curriculum Writers

---

Professor Dr. Muhammad Tahir  
Consultant (Sciences),  
Ministry of Education, ISLAMABAD

Dr. Malik Muhammad Yousaf  
Assistant Professor, Department of Mathematics,  
Quaid-i-Azam University, ISLAMABAD

Mr. Saleem Ullah  
Assistant Director (Academics)/  
Lecturer in Mathematics,  
Federal Directorate of Education,  
ISLAMABAD

Mr. Jamshed Khatak  
Associate Professor/ Vice Principal,  
F. G College of Commerce,  
Sector H-8, ISLAMABAD

### Advisors

---

Professor Dr. Shahid Siddiqui  
Chairman Mathematics Department,  
Punjab University,  
LAHORE

Professor Dr. Muhammad Aslam Noor  
Department of Mathematics,  
COMSATS Institute of Information Technology,  
ISLAMABAD

Professor Dr. Gulzar Ali Khan  
Mathematics Department,  
University of Peshawar,  
PESHAWAR

Dr. Tahir Mehmood  
Chairman Mathematics Departments,  
The Islamia University,  
BAHAWALPUR

Professor Dr. Syed Arif Kamal  
Mathematics Department,  
University of Karachi,  
KARACHI

Professor Dr. Lala Rukh  
Head of Mathematics Department,  
University of Baluchistan,  
QUETTA

Professor Dr. Muhammad Shafiq Baig  
Chairman Mathematics Department,  
University of Azad Jammu & Kashmir,  
MUZAFFARABAD (AJ&K)

Dr. Muhammad Nawaz  
Assistant Professor  
Mathematics Department,  
G C University,  
FAISALABAD

Mr. Abid Saeed Abbasi  
Manager Meezan Bank Limited,  
Sector I-9,  
ISLAMABAD

## Examination Experts

---

Mr. Abdul. Harneed Gill  
Associate Professor (Retd)  
Mathematics Department,  
Government Islamia College,  
Civil Lines, Lahore

Mr. Muhammad Shafique  
Associate Professor (Retd)  
Mathematics Department,  
GC University, Sargodha

Mr. Bashir Muhammad  
Associate Professor  
Mathematics Department,  
Government Degree College,  
Landi Kotel, Khyber Agency (NWFP)

Mr. Muhammad Saeed  
Assistant Professor of Mathematics  
Government Degree College,  
Chapman (Baluchistan)

Mr. Shaukat Iqbal Paracha  
Associate Professor/Vice Principal  
Mathematics Department,  
F.G. Sir Syed College,  
Rawalpindi Cantt

Mr. Muhammad Islam Chohan  
Associate Professor  
Mathematics Department,  
Government College, Gujranwala

Mr. Naeem Akhtar Abbasi  
Assistant Professor of Mathematics  
Govt. Postgraduate College  
Abbottabad (NWFP)

Mr. Muhammad Ayub Abbasi  
Associate Professor of Mathematics  
Govt. Postgraduate College,  
Muzaffarabad (AJ&K)

## Educationists

---

Professor Dr. Shoaib-ud-din  
Mathematics Department,  
Punjab-University Lahore

Professor Dr. M A Rashid  
Centre for Advanced  
Physics (CAMP),  
National University of Sciences and  
Technology (NUST), Rawalpindi

Professor Dr. Muhammad Aslam Butt  
Mathematics Department,  
Lahore University of Management  
Sciences (LUMS), Lahore

Professor Dr. Muhammad Aslam Awan  
Mathematics Department,  
G. C University, Lahore

Professor Dr. Muhammad Rafiq  
Head Mathematics Department,  
EME College,  
National University of Sciences and  
Technology (NUST), Rawalpindi

Dr. Syed Mubashir Ali  
Associate Professor (Finance)  
Suleman Dawood School of Business  
Lahore University of Management  
Sciences (LUMS), Lahore

Professor Sabiha Khalid  
Head of Mathematics Department,  
Govt. Postgraduate College for Women  
Satellite Town, Rawalpindi

Ms. Shama Javaid  
Associate Professor  
Mathematics Department,  
Govt. Postgraduate College for Women  
Satellite Town, Rawalpindi

Ms. Farasat Tahir  
Associate Professor  
Mathematics-Department,  
Govt. Postgraduate College for Women  
Satellite Town, Rawalpindi

Ms. Sajida Anwar  
Associate-Professor  
Mathematics Department,  
Govt. Postgraduate College for Women  
Satellite Town, Rawalpindi

Ms. Nargis Razzaque  
Associate Professor  
Mathematics Department,  
Govt. Postgraduate College for Women  
Satellite Town, Rawalpindi

Ms. Abida Sultana  
Assistant Professor  
Mathematics Department,  
Govt. Postgraduate College for Women  
Satellite Town, Rawalpindi

Ms. Fauzia Anjum  
Lecturer  
Economics Department,  
Govt. Post-graduate College for Women  
Satellite Town, Rawalpindi

Ms. Zainab Javaid  
Assistant Professor  
Statistics Department,  
Govt. Post-graduate College for Women  
Satellite Town, Rawalpindi

Mr. Saeed Akhtar  
Associate Professor  
Mathematics Department,  
Government College, Gujranwala

Ms. Nighat Rasheed  
Associate Professor  
Mathematics Department,  
Govt. Postgraduate College for Women  
Satellite Town, Rawalpindi

Mr. Asif Javaid  
Assistant Professor  
Mathematics Department,  
Government College, Gujranwala

Mr. Abdul Rashid Munhall  
Associate Professor  
Mathematics Department,  
Fauji Foundation College, Rawalpindi

Ms. Razia Siddique  
Associate Professor  
Mathematics Department,  
Govt. Postgraduate College for Women  
Satellite Town, Rawalpindi

Ms. Naheed Jamal  
Assistant Professor  
Mathematics Department,  
Govt. Postgraduate College for Women  
Satellite Town, Rawalpindi

Ms. Safia Shaheen  
Lecturer  
Economics Department,  
Govt. Post-graduate College for Women  
Satellite Town, Rawalpindi